

3.5 keV X-ray Line Signal from Decay of Right-Handed Neutrino due to Transition Magnetic Moment

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We consider the dark matter model with radiative neutrino mass generation where the Standard Model is extended with three right-handed singlet neutrinos (N_1 , N_2 and N_3) and one additional $SU(2)_L$ doublet scalar η . One of the right-handed neutrinos (N_1), being lightest among them, is a leptophilic fermionic dark matter candidate whose stability is ensured by the imposed \mathbb{Z}_2 symmetry on this model. The second lightest right-handed neutrino (N_2) is assumed to be nearly degenerated with the lightest one enhancing the co-annihilation between them. The effective interaction term among the lightest, second lightest right-handed neutrinos and photon containing transition magnetic moment is responsible for the decay of heavier right-handed neutrino to the lightest one and a photon ($N_2 \rightarrow N_1 + \gamma$). This radiative decay of heavier right-handed neutrino with charged scalar and leptons in internal lines could explain the X-ray line signal ~ 3.5 keV recently claimed by XMM-Newton X-ray observatory from different galaxy clusters and Andromeda galaxy (M31). The value of the transition magnetic moment is computed and found to be several orders of magnitude below the current reach of various direct dark matter searches. The other parameter space in this framework in the light of the observed signal is further investigated.

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I. INTRODUCTION

One of the enigmas of modern particle physics is dark matter (DM) which, according to the recent survey of PLANCK [1], consists of $\sim 26.8\%$ of the total energy content of the universe. Various astrophysical and cosmological observations [2–4] strongly suggest convincing hints of the existence of dark matter which is non-relativistic or cold in nature. The particle nature of dark matter is still unknown. The weakly interacting massive particles (WIMPs) are the most promising candidates for cold dark matter.

The experimental techniques for the detection of dark matter for both direct and indirect cases are very challenging. In direct detection experiments, the recoil energy of the target nucleus scattered off by DM particle is measured whereas the signatures of the annihilations or decays of DM particles such as charged particles, photons and neutrinos etc. are aimed to detect in indirect searches. The monochromatic line feature of such decay or annihilation products of DM are particularly significant in predicting the nature of DM particles. A huge variety of DM models in the framework of WIMP scenario with masses of DM spanning from keV to TeVs has been addressed in several literatures and their direct and indirect detection prospects have been widely studied [5–17].

Recently an evidence of X-ray line of energy 3.55 keV with more than 3σ CL has been reported from the analysis of X-ray data of 73 galaxy clusters from XMM-Newton observatory [18]. Another group has also claimed a similar line (3.52 keV X-ray line at 4.4σ CL) from the data of X-ray spectra of Andromeda galaxy (M31) and Perseus cluster [19]. The galaxy clusters are assumed to contain huge amount of DM. Thus the signal may have a possible origin related to DM. The observed line has been explained as decay of sterile neutrino dark matter ($\nu_s \rightarrow \nu + \gamma$) with mass of the sterile neutrino 7.06 ± 0.05 keV and mixing angle $\sin^2(2\theta) = (2.2 - 20) \times 10^{-11}$ [19]. Recently many other interesting ideas have been proposed to explain this line signal to come from DM [20–41].

The neutrino oscillation data [42–45] provide strong evidences for neutrino mass. The non-zero neutrino masses and evidences of DM give hints to the physics beyond the Standard Model (SM). The two beyond SM phenomenon, namely the origin of neutrino masses and the existence of cold dark matter may have a connection. In this work we focus on the simplest framework which invokes this idea of connecting both sectors has been proposed by Ma [46]. In this model the neutrino masses are generated via radiative processes with only the DM

particles in the loop. The right-handed neutrino which can be a possible DM candidate interacts with lepton doublets and hence DM in this scenario is leptophilic in nature. The imposed discrete \mathbb{Z}_2 symmetry on this model not only forbids the tree-level Dirac mass terms but also assure a stable cold DM candidate. Phenomenological prospects for DM in this model have been done in Refs. [47–53]. In this paper we consider the case where the lightest right-handed neutrino (N_1) is the cold DM candidate and the second lightest right-handed neutrino (N_2) is nearly degenerated with the cold DM candidate. This situation provides rich phenomenology in direct detection of such dark matter candidate [54]. Elastic scattering cross section for DM-nucleon interaction is suppressed in this case and inelastic scattering that occurs radiatively dominates. The transition from N_2 to N_1 gives rise to monochromatic photon with energy equal to the mass difference between the lightest and second lightest right-handed neutrinos. If the mass difference between N_2 and N_1 is of \sim keV, then the recent observation of X-ray line can be accommodated in this beyond SM scenario.

The paper is organised as follows. In Sec. I the theoretical framework of the model is briefly discussed. Explanation of the observed X-ray line in this model framework and a study of the constrained parameter space are done in the next section. In Sec. IV a brief summary of this work and some conclusions are drawn.

II. THE MODEL

We consider the model proposed by Ma [46] which is the extension of Standard Model with three gauge singlet right-handed neutrinos N_1 , N_2 , N_3 and an extra $SU(2)_L$ doublet scalar η . The fields can be written as,

$$N_1, \quad N_2, \quad N_3, \quad \eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}. \quad (\text{II.1})$$

The doublet scalar η is assumed to obtain no vacuum expectation value and hence inert. An additional discrete \mathbb{Z}_2 symmetry is imposed on the model. The stability of the cold dark matter candidate in this model is guaranteed by this symmetry. Not only that the tree-level Dirac masses of neutrinos are forbidden for this additional \mathbb{Z}_2 symmetry. SM gauge group and \mathbb{Z}_2 charges of the particles are shown in Tab. I.

Particle	N_k ($k = 1, 2, 3$)	η^0	η^+
$(SU(2)_L, U(1)_Y)$	$(\mathbf{1}, 0)$	$(\mathbf{2}, 1/2)$	$(\mathbf{2}, 1/2)$
\mathbb{Z}_2	odd (-)	odd (-)	odd (-)

TABLE I. Additional fields under SM gauge group and \mathbb{Z}_2 symmetry

The Lagrangian for the right-handed neutrinos, N_k ($k = 1, 2, 3$) invariant under both SM gauge symmetry and \mathbb{Z}_2 symmetry can be written as,

$$\mathcal{L}_N = \overline{N}_i i \not{\partial} P_R N_i + (D_\mu \eta)^\dagger (D^\mu \eta) - \frac{M_i}{2} \overline{N}_i^c P_R N_i + h_{\alpha i} \overline{\ell}_\alpha \eta^\dagger P_R N_i + \text{h.c.}, \quad (\text{II.2})$$

where $h_{\alpha k}$, l_α and M_k represent Yukawa couplings, lepton doublet and the mass of the right-handed neutrino of type k (N_k) respectively. In our following work h_α and M_k are chosen to be real without any loss of generality. The invariant scalar potential containing the Higgs doublet Φ and the additional $SU(2)_L$ doublet η is given by,

$$\begin{aligned} \mathcal{V}(\phi, \eta) = & m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2 \\ & + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\lambda_5}{2} (\phi^\dagger \eta)^2 + \text{h.c.}, \end{aligned} \quad (\text{II.3})$$

The tree-level Dirac mass terms for neutrinos can not be generated since the vacuum expectation value of the doublet η ($\langle \eta \rangle$) to chosen be zero. After electroweak symmetry breaking SM Higgs doublet obtains vacuum expectation value, $v = 246$ GeV and the Majorana masses of neutrinos are generated radiatively via one-loop diagrams with η^0 and N_k in internal lines. The model could explain both possibility of scalar (η^0) and fermion (N_k) as DM. But we choose the mass of one of the three right-handed neutrinos (N_1) is considered to be lightest among the particles added to SM and hence it is a stable candidate of DM. From the forth term of the Lagrangian in Eq. II.2 it is clear that the right-handed neutrino interacts only with the SM lepton doublet and hence leptophilic.

The radiatively generated effective Majorana neutrino masses can be expressed as [46],

$$(m_\nu)_{\alpha\beta} \simeq \sum_{i=1}^3 \frac{2\lambda_5 h_{\alpha i} h_{\beta i} v^2}{(4\pi)^2 M_i} I\left(\frac{M_i^2}{M_\eta^2}\right), \quad (\text{II.4})$$

where $M_\eta^2 \simeq m_\eta^2 + (\lambda_3 + \lambda_4) v^2$, M_i are the masses of η and N_i respectively ¹. The smallness of the mass term is guaranteed by the coupling λ_5 . The factor $I(x)$ can be written as,

$$I(x) = \frac{x}{1-x} \left(1 + \frac{x \log x}{1-x} \right). \quad (\text{II.5})$$

¹ Masses of the real and imaginary parts of η^0 and η^\pm are taken to be degenerated for simplicity

Assuming the mass matrix of Eq. II.4 to be diagonalised using the PMNS matrix which provides very well explanation for the neutrino oscillation data, one can find some conditions imposed on $h_{\alpha i}$ as [49],

$$\begin{aligned} \sum_{k=1}^3 \left(2h_{ek}^2 \sin 2\theta + 2\sqrt{2}h_{ek}(h_{\mu k} - h_{\tau k}) \cos 2\theta - (h_{\tau k} - h_{\mu k})^2 \sin 2\theta \right) &= 0, \\ \sum_{k=1}^3 h_{ek}(h_{\mu k} + h_{\tau k}) &= 0, \quad \sum_{k=1}^3 (h_{\mu k} - h_{\tau k})(h_{\mu k} + h_{\tau k}) = 0. \end{aligned} \quad (\text{II.6})$$

One of the simple solutions for these conditions on $h_{\alpha i}$ (Eq. II.6) is achieved by choosing the flavour structure of $h_{\alpha i}$ as,

$$h_{ei} = 0, \quad h_{\mu i} = h_{\tau i}; \quad h_{ej} \neq 0, \quad h_{\mu j} = -h_{\tau j}, \quad (i \neq j) \quad (\text{II.7})$$

Thus either i or j takes any two values of k (1,2,3). In matrix notation the structure of the chosen Yukawa couplings of Eq. II.7 can be written as,

$$h_{\alpha i} = \begin{pmatrix} 0 & 0 & h'_3 \\ h_1 & h_2 & h_3 \\ h_1 & h_2 & -h_3 \end{pmatrix}. \quad (\text{II.8})$$

The Yukawa couplings of Eq. II.8 imply the values of θ_{12} , θ_{23} and θ_{13} to be $\tan^{-1}(\frac{h'_3}{\sqrt{2}h_3})$, $\pi/4$ and 0 respectively. But from recent observations suggest different values of these mixing angles. Then the structure of the matrix will be slightly modified. The result of this work will not be vastly modified due to such changes.

III. X-RAY LINE IN THIS FRAMEWORK

One of the terms in the Lagrangian of this framework that represents the interaction among the lightest right-handed neutrino (N_1), second lightest right-handed neutrino (N_2) and photon is given by [54],

$$\mathcal{L} = i \left(\frac{\mu_{12}}{2} \right) \overline{N_2} \sigma^{\mu\nu} N_1 F_{\mu\nu}, \quad (\text{III.1})$$

where μ_{12} is the coefficient of this interaction and called *transition magnetic moment* between the right-handed neutrinos, N_1 and N_2 . In the above $F_{\mu\nu}$ is the so-called electromagnetic field tensor. The three-point vertex interaction term of this type is also responsible in

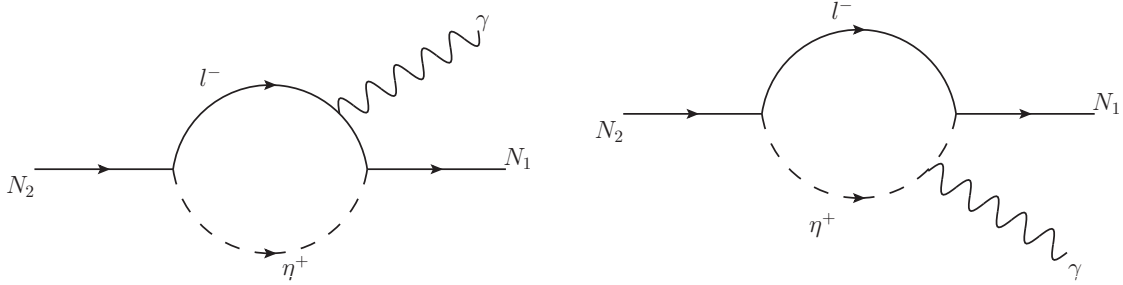


FIG. 1. *Feynman diagrams showing the decay of second lightest right-handed neutrino, N_2 to lightest right-handed neutrino, N_1 and photon (γ) via radiative processes.*

contributing to the inelastic scattering of the right-handed neutrinos with nucleons via 1-loop processes.

The X-ray line appears when there is a transition from the state, N_2 to N_1 . The presence of transition magnetic moment solely triggers such a decay process to occur. The expression of decay width for this process can be written as,

$$\Gamma(N_2 \rightarrow N_1 \gamma) = \frac{\mu_{12}^2}{\pi} \delta^3, \quad (\text{III.2})$$

where $\delta = E_\gamma$ is the energy of the emitted photon which is nothing but the mass difference between the lightest and the second lightest right handed neutrinos present in this framework. The Feynman diagrams responsible for such process are shown in Fig. 1.

The calculated value of the decay width for the decay process of N_2 to N_1 and a photon from the observed X-ray line data is $\sim 1.15 \times 10^{-52}$ GeV [39]. Thus one can find from Eq. III.2 that to comply the observed data for X-ray line with the framework of this model, the absolute value of μ_{12} should be $\sim 2.9 \times 10^{-18}$ GeV $^{-1}$.

The order of the value of $|\mu_{12}|$ is particularly important for studying the prospects of the direct detection of dark matter. The predicted value of $|\mu_{12}|$ from the recently reported X-ray line data is several orders of magnitude below from the current reach of various DM direct detection experiments [54]. As the mass of the dark matter in this model is the lightest right-handed neutrino with heavy mass possibly in the range from few hundreds of GeV to few thousands of GeV, the direct DM searches should probe these massive right-handed neutrinos in this mass range.

The expression for μ_{12} in the present scenario can be written in terms of model parame-

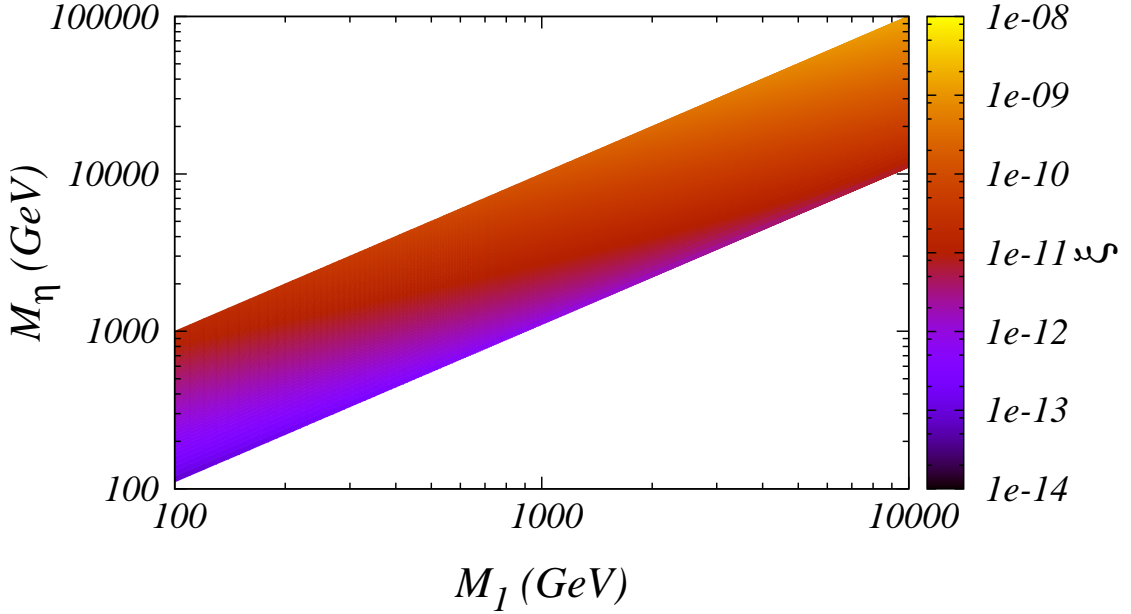


FIG. 2. The allowed parameter space consisting of M_1 , M_η and ξ consistent with the recently reported 3.5 keV X-ray line data. The value of ratio of the mass of N_1 to that of η is chosen to be within 10.0, i.e., $1.0 < M_\eta/M_1 \leq 10.0$ in this plot. The considered range of M_1 is from 10^2 GeV to 10^4 GeV. The phase factor ξ are shown by the colour index where ξ varies from blue coloured region to yellow region as its value increases. See text for more details.

ters [54] as,

$$\mu_{12} = - \sum_{\alpha} \frac{\text{Im}(h_{\alpha 2}^* h_{\alpha 1}) e}{2(4\pi)^2 M_\eta^2} 2M_1 I_m \left(\frac{M_1^2}{M_\eta^2}, \frac{m_\alpha^2}{M_\eta^2} \right), \quad (\text{III.3})$$

m_α is the mass eigenvalue of ordinary neutrino of flavour α , e is the electric charge of proton. The term $\text{Im}(h_{\alpha 2}^* h_{\alpha 1})$ in Eqn. III.3 is related to the phase difference, ξ between the Yukawa couplings $h_{\alpha 2}$ and $h_{\alpha 1}$ for flavour α . For the matrix of Yukawa couplings of Eq. II.8 the value of the factor, $\text{Im}(h_{\alpha 2}^* h_{\alpha 1})$ is zero for one flavour and contributes equally for the remaining flavours. In the above the function I_m comes from loop integral and can be expressed as,

$$I_m(x, y) = - \int_0^1 \frac{z(1-z)}{xz^2 - (1+x-y)z + 1} dz. \quad (\text{III.4})$$

Considering masses of ordinary neutrinos are negligible with respect to that of η , i.e., $m_\alpha \ll M_\eta$, the allowed parameter space for the model parameters, M_1 , M_η and ξ is obtained

from the computed value of $|\mu_{12}|$ from 3.5 keV X-ray line data.² The plot showing the variation of the parameters constrained from observed X-ray line data is shown in Fig. 2. In this plot the ratio (r) of M_η to M_1 is taken to be between 1.0 to 10.0, i.e., $1.0 < M_\eta/M_1 \leq 10.0$. The range of the constrained values of the phase factor, ξ for those mass ratios ($1.0 < r \leq 10.0$) spanning from $\sim 10^{-14}$ to $\sim 10^{-8}$. The situation would have been slightly modified if one incorporate the precise values of mixing angles (for example, non-zero θ_{13}). The Yukawa matrix structure is then modified and the phase factor for each flavour α will be different in general. But it can be shown that for such cases the the order of the sum of the phase factors will be almost of similar order that has been obtained in this case. The phase factor determines the coannihilation of $N_1 - N_2$ and the effective interaction of right-handed neutrino DM with nuclei. The result shows the values of phase factor ξ with much smaller orders for the considered mass range than expected to be give signatures of direct detection. Hence the coannihilation channels and the DM-nuclei interaction is much lowered from the computed value of ξ constrained by the 3.5 keV X-ray line data. Thus the possibility of direct detection of dark matter in this framework is suppressed by few orders from the reach of ongoing direct DM search experiments.

IV. SUMMARY AND CONCLUSION

We have shown that the radiative neutrino mass model can explain the observed 3.5 keV X-ray line signal from the data of various galaxy clusters and Andromeda galaxy (M31). This model can accommodate naturally both neutrino mass and stable cold dark matter candidate. The small mass difference between the lightest and the second lightest right-handed neutrino have been considered to produce the energy of the X-ray signal. Thus the transition from $N_2 \rightarrow N_1 + \gamma$ due to transition magnetic moment via radiative processes involving leptons and charged scalar in internal lines can naturally accommodate all the requirements for the X-ray line signal. The value of the transition magnetic moment (μ_{12}) for such an observed signal is estimated to be few orders of magnitude smaller than the reach of recent DM direct detection experimental limits sustaining the possibility of the cold DM candidate in this model to be detected directly. The other parameters of this model,

² The mass of ordinary neutrino is several orders of magnitude smaller than the mass of doublet scalar η which is few hundreds of GeV or more in this framework and hence the ratio, $\frac{m_\alpha}{M_\eta}$ is $\ll 1$

namely masses of lightest right-handed neutrino (N_1), doublet scalar (η) and phase factor (ξ) between Yukawa couplings, h_1 and h_2 are further constrained from the observed X-ray line data. A very small but non-zero value of the phase difference between Yukawa couplings, h_1 and h_2 have been predicted. Also the coannihilation between N_1 and N_2 is reduced and the s-wave contribution of dark matter annihilation cross section is calculated to be reduced. Finally the analysis performed here for this model framework would be viable for any DM signal in this energy regime. In addition the dark matter candidate (lightest right-handed neutrino), being leptophilic and massive, can potentially explain AMS-02 positron excess.

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